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CONTRA bT^μ- CONTINUOUS FUNCTION IN SUPRA TOPOLOGICAL SPACES

K.Krishnaveni*, M.Vigneshwaran
* Department of Mathematics Kongunadu Arts and Science College Comibatore.
Department of Mathematics Kongunadu Arts and Science College Comibatore.

ABSTRACT
In this paper, we introduce the concept of contra bT^μ- continuous functions and contra bT^μ-irresolute. We obtain the basic properties and their relationship with other forms of contra supra continuous functions in supra topological spaces.

KEYWORDS: contra bT^μ- continuous function ,contra bT^μ-irresolute, almost contra bT^μ- continuous function and perfect contra bT^μ-irresolute.

INTRODUCTION
In 1983 Mashhour et al [6] introduced Supra topological spaces and studied S- continuous maps and S^*-continuous maps. In 1996, Dontchev[3] presented a new notation of continuous function called contra- continuity in topological spaces.

The purpose of this paper is to introduce the concept of contra bT^μ- continuous functions and contra bT^μ-irresolute and studied its basic properties. Also we defined almost contra bT^μ- continuous functions and perfect contra bT^μ-irresolute function and investigated their relationship to other functions in supra topological spaces.

PRELIMINARIES
Definition 2.1[6,8] A subfamily of μ of X is said to be a supra topology on X, if

(i) $X, \phi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 2.2[8]

(i) The supra closure of a set A is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap\{B : B \text{ is a supra closed set and } A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup\{B : B \text{ is a supra open set and } A \supseteq B\}$.

Definition 2.3[6] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ be a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4[8] Let (X, μ) be a supra topological space. A set A is called a supra b-open set if $A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A))$. The complement of a supra b-open set is called a supra b-closed set.

Definition 2.5[5] A subset A of a supra topological space (X, μ) is called bT^μ-closed set if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is T^μ- open in (X, μ).

Definition 2.6[5] Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function **f** : (X, τ) → (Y, σ) is called bT^{μ}- Continuous if f^{-1} (V) is bT^{μ}- closed in (X, τ) for every closed set V of
(Y, σ).

Definition 2.7[5] Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function **f** : (X, τ) → (Y, σ) is called bT^{μ}- irresolute if f^{-1}(V) is bT^{μ}-closed in (X, τ) for every bT^{μ}-closed set V of (Y, σ).

Definition 2.8[3] Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function **f** : (X, τ) → (Y, σ) is called contra Continuous if f^{-1} (V) is supra closed in (X, τ) for every supra open set V of
(Y, σ).

Definition 2.9[5] A supra topological space (X,μ) is called $_{bT}T_c^{\mu}$-space. If every bT^{μ}-closed set is supra closed set.

CONTRA bT^{μ}- CONTINUOUS FUNCTION

Definition 3.1 A function **f** : (X, τ) → (Y, σ) is called contra bT^{μ}- continuous functions if f^{-1}(V) is bT^{μ}-closed in (X, τ) for every supra open set V of (Y, σ).

Example 3.2 Let X = Y = {a,b,c} with τ = {X,φ,{b},{a,b}} and σ = { Y, φ,{a}}.
Let **f** : (X, τ) → (Y, σ) be the identity function . Here f is contra bT^{μ}- continuous functions.

Example 3.3 Let X = Y = {a,b,c} with τ = {X,φ,{a} Let **f** : (X, τ) → (X, τ) be the identity function . Here f is not contra bT^{μ}- continuous functions. Since V = {a} is supra open set in (Y, σ), f^{-1}({a}) = {a} is not in bT^{μ}- closed set in (X, τ).

Theorem 3.4 Every contra continuous function is contra bT^{μ}- continuous.

Proof Let f: X→ Y be contra continuous. Let V be any supra open in Y. Then the inverse image f^{-1}(V) is supra closed in X. Since every supra closed is bT^{μ}- closed, f^{-1}(V) is bT^{μ}- closed in X. Therefore f is contra bT^{μ}- continuous.

Remark 3.5 The converse of the above theorem is not true and it is shown by the following example.

Example 3.6 Let X = Y = {a,b,c} with τ = {X,φ,{b},{a,b}} and σ = { Y, φ,{a}}.
Let **f** : (X, τ) → (Y, σ) be the identity function . Here f is contra bT^{μ}- continuous functions and not contra continuous . Since V = {a} is supra open set in (Y, σ), f^{-1}({a}) = {a} is not supra closed in (X, τ).

Remark 3.7 The composition of two contra bT^{μ}- continuous map need not be contra bT^{μ}- continuous. Let us prove the remark by the following example.

Example 3.8 Let X= Y = {a,b,c}. Let τ = {X,φ,{b},{a,b}} , σ = { Y, φ,{a}} and γ = { Z, φ, {b},{a,b}} . Let f: (X,τ) → (Y,σ) and g: (Y,σ) →(Z, γ). Define f(a) = a, f(b) = b , f(c) = c and g(a) = c, g(b) = b, g(c) = a. Both f and g are contra bT^{μ}- continuous. Define gof: (X,τ) → (Z, γ). Hence {b} is a supra open set of (Z, γ). Therefore $(gof)^{-1}$({b}) = $g^{-1}(f^{-1}$({b })) = g^{-1}({b}) = {b} is not a bT^{μ}- closed set of (X,τ). Hence g o f is not contra bT^{μ}- continuous

Theorem 3.9 If f:(X, τ) →(Y, σ) is contra bT^{μ}- continuous function and g: (Y, σ) → (Z, γ) is supra continuous function then composition gof is contra bT^{μ}- continuous function.

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Proof Let V be supra open set in Z. Since g is supra continuous, then $g^{-1}(V)$ is supra open in Y. Since f is contra bT^{μ}- continuous function, then $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is bT^{μ}- closed in X. Therefore gof is contra bT^{μ}- continuous function.

Theorem 3.10 If $f:(X, \tau) \to (Y, \sigma)$ is bT^{μ}- irresolute function and $g: (Y, \sigma) \to (Z, \gamma)$ is contra bT^{μ}- continuous function then composition gof is contra bT^{μ}- continuous function.

Proof Let V be supra open set in Z. Since g is contra bT^{μ}- continuous function, then $g^{-1}(V)$ is bT^{μ}- closed in Y. Since f is bT^{μ}- irresolute function, then $f^{-1}(g^{-1}(V))$ is bT^{μ}- closed in X. Therefore gof is contra bT^{μ}- continuous function.

Remark 3.11 The concept of bT^{μ}- continuity and contra bT^{μ}- continuity are independent as shown in the following example

Example 3.12 Let $X=Y=\{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}\}$, $\sigma = \{Y, \varphi, \{a\}, \{a,b\}\}$. $f:(X, \tau) \to (Y, \sigma)$ be the identity function . Here f is bT^{μ}- continuous but not contra bT^{μ}- - continuous function, since V={a} is supra open set in Y but $f^{-1}(\{a\}) = \{a\}$ is not bT^{μ}- closed set in X.

Let $X=Y=\{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}\}$, $\sigma = \{Y, \varphi, \{a\},\{b\},\{a,b\}\}$.$f:(X, \tau) \to (Y, \sigma)$ be the function defined by f(a)=c, f(b)=b, f(c)=a. Here f is contra bT^{μ}- continuous but not bT^{μ}- continuous function, since V={b,c} is supra closed set in Y but $f^{-1}(\{b,c\}) = \{a,b\}$ is not bT^{μ}- closed set in X.

Theorem 3.13 If $f:(X, \tau) \to (Y, \sigma)$ is contra bT^{μ}- continuous function and X is ${}_{bT}T_c{}^{\mu}$-space, then f is contra supra continuous.

Proof Let V be supra open set in Y. Since f is contra bT^{μ}- continuous function, then $f^{-1}(V)$ is bT^{μ}- closed in X. Since X is ${}_{bT}T_c{}^{\mu}$-space, We have every bT^{μ}- closed set is supra closed in X, then $f^{-1}(V)$ is supra closed in X. Therefore f is contra supra continuous function.

Definition 3.14 A map $f:(X, \tau) \to (Y, \sigma)$ is called almost contra bT^{μ}- continuous function if $f^{-1}(V)$ is bT^{μ}- closed in (X, τ) for every supra regular open set V in (Y, σ).

Theorem 3.15 Every contra supra continuous function is almost contra bT^{μ}- continuous function.

Proof Let $f:(X, \tau) \to (Y, \sigma)$ be a contra supra continuous function. Let V be a supra regular open set in (Y, σ). We know that every supra regular open set is supra open, then V is supra open in (Y, σ). Since f is contra supra continuous function, $f^{-1}(V)$ is supra closed in (X, τ). We know that every supra closed set is bT^{μ}- closed, implies $f^{-1}(V)$ is bT^{μ}- closed in (X, τ). Therefore f is almost contra bT^{μ}- continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.16 Let $X=Y=\{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}\}$, $\sigma = \{Y, \varphi, \{a\},\{a,b\}\}$.$f:(X, \tau) \to (Y, \sigma)$ be the identity function . Here f is almost contra bT^{μ}- continuous, but it is not contra supra continuous, Since V={a} is supra open in Y but $f^{-1}(\{a\}) = \{a\}$ is not supra closed in X.

Theorem 3.17 Every contra bT^{μ}- continuous function is almost contra bT^{μ}- continuous function.

Proof Let $f:(X, \tau) \to (Y, \sigma)$ be a contra bT^{μ}- continuous function. Let V be a supra regular open set in (Y, σ). We know that every supra regular open set is supra open, then V is supra open in (Y, σ). Since f is contra bT^{μ}- continuous function, $f^{-1}(V)$ is bT^{μ}- closed in (X, τ). Therefore f is almost contra bT^{μ}- continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.18 Let X=Y={a, b, c} and τ = {X, φ, {a}} , σ = {Y, φ, {a},{a,b}}.f:(X, τ) $\rightarrow$ (Y, σ) be the identity function . Here f is almost contra bT^{μ}- continuous, but it is not contra bT^{μ}- continuous, Since V={a} is supra open in Y but f^{-1}({a}) = {a} is not bT^{μ}-closed in X.

Theorem 3.19 If a map f: X$\rightarrow$Y from supra topological space X into a supra topological space Y. The following statement are equivalent .

(a) f is almost contra bT^{μ}- continuous.
(b) For every supra regular closed set F of Y , f^{-1}(F) is bT^{μ}- open in X.

Proof (a) $\Rightarrow$ (b)
Let F be a supra regular closed set in Y, then Y-F is a super regular open set in Y.
By (a) f^{-1}(Y-F) = X – f^{-1}(F) is bT^{μ}- closed set in X. This implies f^{-1}(F) is bT^{μ}- open set in X. Therefore (b) holds.
(b)$\Rightarrow$(a)

Let G be a supra regular open set of Y. The Y-G is supra regular closed set of Y. By (b) f^{-1}(Y-G) is bT^{μ}- open in X. This implies Y- f^{-1}(G) is bT^{μ} - open in X, which implies f^{-1}(G) is bT^{μ}-closed set in X. Therefore (a) holds.

Definition 3.20 A Space (X, τ) is bT^{μ}- locally indiscrete if every bT^{μ}- open (bT^{μ}- closed) set is supra closed(supra open) in (X, τ).

Theorem 3.21 If f:(X, τ) $\rightarrow$ (Y, σ) is bT^{μ}- continuous function and X is bT^{μ}- locally indiscrete, then f is contra bT^{μ}- continuous.

Proof Let V be supra open set in Y. Since f is bT^{μ}- continuous function, then f^{-1}(V) is bT^{μ}- open in X. Since X is bT^{μ}- locally indiscrete, then f^{-1}(V) is supra closed set in X. We know that every supra closed set is bT^{μ} - closed set . Therefore f^{-1}(V) is bT^{μ}- closed set in X. Hence f is contra bT^{μ}-continuous function.

Theorem 3.22 If f:(X, τ) $\rightarrow$ (Y, σ) is a surjective bT^{μ}-irresolute and g: (Y, σ) $\rightarrow$ (Z, γ) be any function such that gof: (X, τ) $\rightarrow$ (Z, γ) is contra bT^{μ}-continuous function, iff g is contra bT^{μ}-continuous.

Proof
Suppose gof is contra bT^{μ}-continuous, Let V be a supra closed set in Z, then $(gof)^{-1}$(V) = $f^{-1}(g^{-1}$(V)) is bT^{μ}-open in (X, τ). Since f is surjective and bT^{μ}-irresolute , then f$(gof)^{-1}$(V)=f($f^{-1}(g^{-1}$(V))) = g^{-1}(V) is supra N-open in (Y, σ).Hence g is contra bT^{μ}-continuous function.

Conversely, Suppose g is contra bT^{μ}-continuous, Let V be supra closed in Z, then g^{-1}(V) is bT^{μ}-open in Y. Since f is surjective and bT^{μ}-irresolute, then $f^{-1}(g^{-1}$(V)) is bT^{μ}-open in X. Hence gof is contra bT^{μ}-continuous function.

Theorem 3.23 If f:(X, τ) $\rightarrow$ (Y, σ) is a bT^{μ}-continuous and g: (Y, σ) $\rightarrow$ (Z, γ) is contra bT^{μ}-continuous function and (Y, σ) is ${}_{bT}T_c{}^{\mu}$-space, then gof: (X, τ) $\rightarrow$ (Z, γ) is contra bT^{μ}-continuous function.
Proof Let V be any supra open set in Z, then g^{-1}(V) is bT^{μ}-closed set in Y. since Y is ${}_{bT}T_c{}^{\mu}$-space, g^{-1}(V) is supra closed set in Y. Since f is bT^{μ}-continuous $f^{-1}(g^{-1}$(V)) = $(gof)^{-1}$(V) is bT^{μ}-closed set in X. Hence gof is contra bT^{μ}-continuous.

CONTRA BT^{μ}- IRRESOLUTE FUNCTION

Definition 4.1 A function f:(X,τ)$\rightarrow$(Y,σ) is called contra bT^{μ}- irresolute function if f^{-1}(V) is bT^{μ}- closed in (X,τ) for every bT^{μ}- open set V in (Y,σ).

Definition 4.2 A function f:(X,τ)$\rightarrow$(Y,σ) is called perfectly contra bT^{μ}- irresolute function if f^{-1}(V) is bT^{μ}- closed and bT^{μ}- open in (X,τ) for every bT^{μ}- open set V in (Y,σ).

Theorem 4.3 Every contra bT^{μ}- irresolute function is contra bT^{μ}- continuous.

Proof Let f: $(X,\tau)\to(Y,\sigma)$ be a contra bT^{μ}- irresolute function. Let V be a supra open set in (Y,σ). We know that every supra open set is bT^{μ}- open set , then V is bT^{μ}- open in (Y,σ). Since f is contra bT^{μ}- irresolute function, $f^{-1}(V)$ is bT^{μ}- closed in (X,τ). Therefore f is contra bT^{μ}- continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.4 Let X=Y= {a,b,c}, τ = {X,ϕ, {a}} , σ = { Y,ϕ,{a},{b},{a,b}}.A function f: $(X,\tau)\to(Y,\sigma)$ is defined by f(a) = c, f(b) = b, f(c) = a. Here f is contra bT^{μ}- continuous but not contra bT^{μ}- irresolute. Since V = {b,c} is bT^{μ}- open set in (Y,σ) and f^{-1}({b,c})= {a,b} is not in bT^{μ}- closed set in (X,τ).

Theorem 4.5 If f: $(X,\tau)\to(Y,\sigma)$ is a bT^{μ}- irresolute and g: $(Y,\sigma)\to(Z,\gamma)$ is contra bT^{μ}- irresolute function, then gof : $(X,\tau)\to(Z,\gamma)$ is contra bT^{μ}- irresolute function.

Proof Let V be any bT^{μ}- open set in Z. Since g is contra bT^{μ}- irresolute then g^{-1} (V) is bT^{μ}- closed set in Y. Since f is bT^{μ}- irresolute $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is bT^{μ}- closed set in X. Hence gof is contra bT^{μ}- irresolute function.

Theorem 4.6 If f: $(X,\tau)\to(Y,\sigma)$ is a contra bT^{μ}- irresolute and g: $(Y,\sigma)\to(Z,\gamma)$ is bT^{μ}- irresolute function, then gof : $(X,\tau)\to(Z,\gamma)$ is contra bT^{μ}- irresolute function.

Proof Let V be any bT^{μ}- open set in Z. Since g is bT^{μ}- irresolute then g^{-1} (V) is bT^{μ}- open set in Y. Since f is contra bT^{μ}- irresolute $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is bT^{μ}- closed set in X. Hence gof is contra bT^{μ}- irresolute function.

Theorem 4.7 Every perfectly contra bT^{μ}- irresolute is contra bT^{μ}- irresolute function.

Proof Let f: $(X,\tau)\to(Y,\sigma)$ be a perfectly contra bT^{μ}- irresolute function. Let V be a bT^{μ}- open set in (Y,σ). Since f is perfectly contra bT^{μ}- irresolute function, $f^{-1}(V)$ is bT^{μ}- closed and bT^{μ}- open in (X,τ). Therefore f is contra bT^{μ}- irresolute function .

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.8 Let X=Y= {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}, σ = {Y,ϕ,{b},{b,c},{a,b}} f: $(X,\tau)\to(Y,\sigma)$ be a function defined by f(a) = a, f(b) = c, f(c) = b. Here f is contra bT^{μ}- irresolute function but not perfectly contra bT^{μ}- irresolute function. Since V = {a,c} is bT^{μ}- open set in (Y,σ) and f^{-1}({a,c}) = {a,b} is not bT^{μ}- closed and bT^{μ}- open set in (X,τ)

Theorem 4.9 Every perfectly contra bT^{μ}- irresolute is contra bT^{μ}- irresolute function.
Proof Let f: $(X,\tau)\to(Y,\sigma)$ be a perfectly contra bT^{μ}- irresolute function. Let V be a bT^{μ}- open set in (Y,σ). Since f is perfectly contra bT^{μ}- irresolute function, $f^{-1}(V)$ is bT^{μ}- closed and bT^{μ}- open in (X,τ). Therefore f is bT^{μ}- irresolute function .

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.10 Let X=Y= {a,b,c} and τ = {X,ϕ,{a}}, σ = {Y,ϕ,{a},{a,b}} f: $(X,\tau)\to(Y,\sigma)$ be a identity function. Here f is bT^{μ}- irresolute function but not perfectly contra bT^{μ}- irresolute function. Since V = {a,c} is bT^{μ}- open set in (Y,σ) and f^{-1}({a,c}) = {a,c} is not bT^{μ}- closed and bT^{μ}- open set in (X,τ).

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